



Modeling crowd evacuation of a building based on seven methodological approaches

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ABSTRACT

Crowd evacuation of a building has been studied over the last decades. In this paper, seven methodological approaches for crowd evacuation have been identified. These approaches include cellular automata models, lattice gas models, social force models, fluid-dynamic models, agent-based models, game theoretic models, and approaches based on experiments with animals. According to available literatures, we discuss the advantages and disadvantages of these approaches, and conclude that a variety of different kinds of approaches should be combined to study crowd evacuation. Psychological and physiological elements affecting individual and collective behaviors should be also incorporated into the evacuation models.

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1. Introduction

Accidents (e.g., fire, bomb threat or toxic gas release) in a building (e.g., a theatre, a stadium, a shopping mall or an air terminal) always threaten human lives. Evacuation of a group of pedestrians from the hazardous areas, under temporal and physical constraints, is a major issue. If the crowd fails to escape from a building in time, due to failure of obstacles' avoidance or wrong exit selection, people may be injured and killed (by fire, bomb or toxic gas). Also, the crowd's behavior (e.g., simultaneously rushing towards the exits, shuffling, pushing, crushing, and trampling) itself may result in injuries and death. Until now, many of the phenomena and laws emerging from the interactions among evacuees and their environments have been characterized only through modeling, due to the absence of data from real evacuations [1]. Currently, evacuation simulation models, such as BGRAF [2,3], EXODUS [2–4], SIMULEX [2–4], can be used to predict the performance of evacuations in a specific building and thus become an important tool for doing the building evacuation analysis.

To aid model users to select the appropriate model for specific projects, some work has been done. Friedman [5] reviewed egress models. Olenick and Carpenter [6] reported that approximately four times as many evacuation models have been identified in their survey than were identified by Friedman (1992). Furthermore, Gwynne et al. [2] classified 22 evacuation models based on three different approaches, namely, optimization, simulation, and risk assessment. In recent years, Kuligowski [3] has classified 28

different egress models based on the level of complexity in occupant behavior. Santos and Aguirre [4] have described a critical review of emergency evacuation simulation models from the simulation methods including flow-based, cellular automata, and agent-based models (ABMs). These models have become the software tools. For example, BGRAF can simulate any type of building [3], and EXITT specializes in residences [3].

However, this work is different from what Friedman, Olenick and Carpenter, Gwynne et al., Kuligowski, and Santos and Aguirre did. It summarizes systematically crowd evacuation models based on seven methodological approaches. These approaches include typical models, such as cellular automata models, lattice gas models, social force models, and fluid-dynamic models, as well as innovative models such as ABMs and game theoretic models. Furthermore, approaches based on experiments with animals (e.g., experiments with rats or ants) have been developed to determine evacuation dynamics. It is useful to understand the approaches for crowd evacuation. Moreover, these approaches can be further developed for a more realistic evacuation simulation.

In the following section, there is an outline of studies on crowd evacuation based on the seven approaches. Various scenarios reproduced by these approaches are summarized. Furthermore, the advantages and disadvantages of these approaches are discussed. Finally, we draw some conclusions and propose further research.

2. Crowd evacuation study approaches

In recent years, researchers have applied the above seven approaches, separately and in combination, to study crowd

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evacuation under various situations. In this section, crowd evacuation models are systematically reviewed and categorized according to six characteristic features, which are described in Table 1.

2.1. Cellular automata models

Cellular automata were first proposed by Von Neumann. Cellular automata are discrete dynamic systems consisting of a regular grid of cells. Cellular automata evolve at each discrete time step, with the value of the variable at one cell determined by the values of variables at the neighboring cells. The variables at each cell are simultaneously updated based on the values of the variables in their neighborhood at the previous time step and according to a set of local rules [7]. At present, cellular automata have been successfully applied to various complex systems, including traffic models and biological fields. Over the last decade or so, cellular automata models have been applied to describe pedestrian dynamics during evacuations. The models divide the space in a uniform grid (Fig. 1).

In recent years, cellular automata models have been developed to study crowd evacuation under various situations. These models can be classified into two categories. The first one is based on the interactions between environments and pedestrians. For example, Zhao et al. [8] proposed a two-dimensional cellular automata random model to study the exit dynamics of occupant evacuation; Perez et al. [9] described a cellular automata model to study the exit dynamics of pedestrians in a room, and investigated the

throughput behavior under different exit conditions for a room; Varas et al. [10] used a two-dimensional cellular automata model to simulate evacuation process from a room with and without obstacles and study the effect of obstacles; Yu and Song [11] proposed a cellular automata model without step back to simulate pedestrian counter flow in a channel considering the surrounding environment. These models demonstrate that various environments such as exit width and obstacles impact on the pedestrian movement.

The other is based on the interaction among pedestrians. Firstly, friction effects of pedestrian behavior have been studied by some cellular automata models. For example, Kirchner et al. [12] proposed an extended stochastic cellular automata model to simulate friction effects and clogging during the evacuation of a large room with one door; Kirchner et al. [13] proposed a cellular automata model for pedestrian dynamics with friction to simulate competitive egress behavior; Schultz et al. [14] presented an enhanced cellular automata model considering repulsion potentials, friction effects, and path finding/guidance algorithms to simulate passenger motion behavior in airport. Secondly, the bi-direction pedestrian behavior has been simulated. For instance, Fang et al. [15] proposed a cellular automata model based on the human judgment to simulate the bi-direction pedestrian movement and study the effect of back stepping on the critical density of phase transition; Li et al. [16] presented a cellular automata model to simulate the bi-direction pedestrian movement in a corridor. Thirdly, herding behavior has been studied. Nishinari et al. [17] proposed a stochastic cellular automata model to investigate pedestrians following each other during an evacuation. Georgoudas et al. [18] used a computational intelligent technique-based cellular automata to simulate pedestrian dynamics during the evacuation of large areas and reproduce some phenomena of crowd dynamics such as clogging and mass behavior; Kirchner and Schadschneider [19] proposed a bionics-inspired cellular automata model to describe the interaction among the pedestrians, and simulate the evacuation from a large room with one or two doors; it is found that for achieving optimal evacuation times a proper combination of herding behavior and use of knowledge about the surrounding is necessary. Additionally, Weng et al. [20] proposed a cellular automata model without step back to simulate pedestrian counter flow with different walk velocities; Yuan and Tan [21] applied a two-dimensional basic cellular automata model based on human behavior (inertial effect, group effect, and unadventurous effect) to investigate the evacuation from a smoke-filled room with multiple exits. Crowd behaviors are so complicated that these models can only show one aspect of human characteristics during crowd evacuation.

Furthermore, cellular automata models have been combined with other approaches to investigate evacuation process. Yang et al. [1] proposed a two-dimensional cellular automata model based on the social force to simulate the effects of kin behavior. Song et al. [22] proposed a cellular automata model entitled CAFE (cellular automata with forces essentials) to simulate evacuation process in a single-exit room. Yamamoto et al. [23] present a real-coded cellular automata (RCA) model based on real-coded lattice gas to simulate the evacuation from a room with an exit of various widths.

For comparison, Table 2 shows features of these cellular automata models.

2.2. Lattice gas models

Lattice gases are a special case of cellular automata, and popularized in the 1980s by Fredkin and Toffoli [24] and by Wolfram [7,25]. In lattice gas models, each pedestrian is

Table 1
Six items of features describing the evacuation models (approaches)

Items of features	Descriptions
Approaches	Seven modeling approaches are applied to study crowd evacuation separately and in combination. They are approaches based on <i>cellular automata (CA)</i> , <i>lattice gas (LG)</i> , <i>social force (SF)</i> , <i>fluid dynamics (FD)</i> , <i>agent-based (AB)</i> , <i>game theory</i> and <i>experiments with animals</i> .
Individuals/groups	In some models (approaches), pedestrians are ideally considered as <i>homogeneous</i> individuals. However, in others, pedestrians are looked on as <i>heterogeneous</i> individuals (groups) by the difference of characteristics (e.g., gender, age, psychology).
Scale	In some models (approaches), where collective phenomena emerge from the complex interactions between many individuals (self-organizing effects), pedestrian dynamics is modeled on a <i>microscopic</i> scale. In others, when a crowd of pedestrians is considered as a whole, pedestrian dynamics is modeled on a <i>macroscopic</i> scale.
Space and time (SAT)	Some modeling approaches are <i>discrete</i> in space and time; the others are <i>continuous</i> .
Situations	Crowd movement is described in <i>normal</i> and <i>emergency</i> situations.
Typical phenomena	Different behaviors can be reproduced in the pedestrian flow simulations.

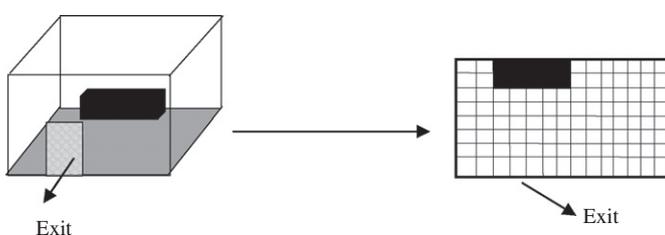


Fig. 1. 3D environment and its corresponding grid of cells. Walls and other fixed obstacles are black; the white cells are areas that can be occupied by pedestrians.

Table 2
Features of these cellular automata models

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Typical phenomena
[1]	CA and SF	Homogeneous	Microscopic	Discrete	Emergency	Kin behavior
[8]	CA	Homogeneous	Microscopic	Discrete	Emergency	Arching
[9]	CA	Homogeneous	Microscopic	Discrete	Emergency	Arching
[10]	CA	Homogeneous	Microscopic	Discrete	Emergency	Effect of obstacles
[11]	CA	Homogeneous	Microscopic	Discrete	Normal	Jamming
[12]	CA	Homogeneous	Microscopic	Discrete	Emergency	Friction effects, clogging
[13]	CA	Homogeneous	Microscopic	Discrete	Emergency	Competitive egress behavior
[14]	CA	Heterogeneous	Microscopic	Discrete	Emergency	Friction effects, repulsion behavior, path finding
[15]	CA	Homogeneous	Microscopic	Discrete	Normal	Jamming, bi-direction movement
[16]	CA	Homogeneous	Microscopic	Discrete	Normal	Side-stepping, backwardness
[17]	CA	Homogeneous	Microscopic	Discrete	Emergency	Following behavior
[18]	CA	Heterogeneous	Microscopic	Discrete	Emergency	Clogging and mass behavior
[19]	CA	Homogeneous	Microscopic	Discrete	Emergency	Herding behavior
[20]	CA	Homogeneous	Microscopic	Discrete	Normal	Lane formation, jamming
[21]	CA	Homogeneous	Microscopic	Discrete	Emergency	Unadventurous, inertial and group effects
[22]	CA and SF	Homogeneous	Microscopic	Discrete	Emergency	Arching, clogging, faster-is-slower
[23]	CA and LG	Homogeneous	Microscopic	Discrete	Normal	Clogging

Note:

Kin behavior: for example, the family members usually insist on gathering together during an emergent evacuation and sometimes there is even a “backtracking” phenomenon.

Faster-is-slower: the faster the people wish to move the slower they can evacuate the room.

Unadventurous effects: most people use the familiar exits.

Inertial effects: once people move towards a certain exit, they usually continue heading the same direction.

Group effects: people help each other in an emergency.

Table 3
Features of these lattice gas models

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Typical phenomena
[26]	LG	Homogeneous	Microscopic	Discrete	Normal	Free flow, choking flow
[27]	LG	Homogeneous	Microscopic	Discrete	Normal	Free flow, choking flow
[28]	LG	Homogeneous	Microscopic	Discrete	Normal	Free flow, choking flow
[29]	LG	Homogeneous	Microscopic	Discrete	Emergency	Counterflow of people crawling on all fours
[30]	LG	Homogeneous	Microscopic	Discrete	Normal/emergency	Side effect, counterflow
[31]	LG	Homogeneous	Microscopic	Discrete	Normal	Jamming
[32]	LG	Homogeneous	Microscopic	Discrete	Emergency	–
[33]	LG	Homogeneous	Microscopic	Discrete	Emergency	Walk and crawl
[34]	LG	Homogeneous	Microscopic	Discrete	Emergency	Jamming (queuing)
[35]	LG	Homogeneous	Microscopic	Discrete	Emergency	Behavior of blind people
[36]	LG	Homogeneous	Microscopic	Discrete	Emergency	Jamming, herding behavior
[37]	LG and SF	Homogeneous	Microscopic	Discrete	Emergency	Arching, clogging

considered as an active particle on the grid. These models are often used to study the features of a crowd of pedestrians by means of probability and statistics.

Lattice gas models have been applied to study the characteristics of pedestrian flow in different building structures. Lattice gas models of pedestrian flow have been applied to simulate crowd flow going outside a hall [26], Lattice gas models of biased-random walkers have been proposed to simulate pedestrian channel flow at a bottleneck [27], to simulate pedestrian flow in the T-shaped channel [28], to simulate the counterflow of crawlers on all fours in the channel [29], to study the side effect (a pedestrian edges through the crowd) on pedestrian counter flow in the channel [30], to simulate shifting of the audience in a hall through a gate [31], to simulate evacuation process through an exit from a hall [32], to simulate the evacuation of walkers and crawlers from a corridor with an exit [33], to simulate evacuation process from a classroom [34]. What is more, lattice gas models have been applied to study crowd evacuation under different situations. For example, lattice gas model of biased-random walkers have been used to simulate evacuation process from a dark room with a few exits [35], to simulate evacuation process from a smoky room [36].

Moreover, lattice gas models have been combined with other approaches to investigate evacuation process. Song et al. [37] proposed the multi-grid model introduced with the force concept of a social force model to simulate interaction forces among pedestrians and those between pedestrians and construction during an evacuation. Additionally, the RCA model based on RLG (depicted in Section 2.1) [23] is another combined evacuation model.

For comparison, Table 3 shows features of these lattice gas models.

2.3. Social force models

In 1995, Helbing and Molnar [38] proposed a social force model for pedestrian motion. The motion of pedestrian α is determined by the following main effects: (1) he/she wants to reach a certain destination; (2) he/she keeps a certain distance from other pedestrians; (3) he/she also keeps a certain distance from borders of obstacles such as walls; (4) he/she is sometimes attracted by other persons (e.g., friends) or objects (e.g., window displays). The total effect of pedestrian

α is determined by

$$\begin{aligned} \underbrace{\vec{F}_\alpha(t)}_{\text{Total Effect}} &= \underbrace{\vec{F}_\alpha^0(\vec{v}_\alpha, v_\alpha^0 \vec{e}_\alpha)}_{\text{Effect of Pedestrian's Desire}} + \underbrace{\sum_\beta \vec{F}_{\alpha\beta}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}_\beta)}_{\text{Repulsive of Pedestrian } \beta} \\ &+ \underbrace{\sum_B \vec{F}_{\alpha B}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}_B)}_{\text{Repulsive of Border B}} + \underbrace{\sum_i \vec{F}_{\alpha i}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}_i, t)}_{\text{Attractive Effective}} \end{aligned}$$

where \vec{v}_α is the actual velocity of pedestrian α ; v_α^0 , the certain desired speed of pedestrian α ; \vec{e}_α , the desired motion direction of pedestrian α ; \vec{r}_α , the actual position of pedestrian α at time t ; \vec{r}_β , the actual position of pedestrian β at time t and \vec{r}_B is the location of border (e.g., wall) B that is nearest to pedestrian α .

If random variations of the pedestrian behavior are taken into account, a fluctuation term is added into the above equation. Then the social force model is defined by

$$\frac{d\vec{w}_\alpha(t)}{dt} = \vec{F}_\alpha(t) + \text{fluctuations},$$

where \vec{w}_α is the preferred velocity.

In 2000, Helbing et al. [39] developed another social force model to simulate panic situations. The model describes human crowd behavior with a mixture of socio-psychological and physical forces. The social force model is determined by the acceleration equation:

$$\underbrace{m_i \frac{dv_i}{dt}}_{\text{Acceleration}} = m_i \underbrace{v_i^0(t) \mathbf{e}_i^0(t) - v_i(t)}_{\text{Driving Force}} + \underbrace{\sum_{j(\neq i)} f_{ij} + \sum_W f_{iW}}_{\text{Interaction Forces}}$$

where m_i is the mass of pedestrian i , v_i , the instantaneous velocity of pedestrian i , v_i^0 , the certain desired speed of pedestrian i , \mathbf{e}_i^0 , the certain direction of pedestrian i , τ_i , a certain characteristic time interval, f_{ij} , pedestrian i tries to keep a distance from other pedestrians j using interaction forces, and f_{iW} is pedestrian i trying to keep a distance from the walls W using interaction forces.

In recent years, social force models have attracted great attention from some researchers and have been further developed to study crowd evacuation. Zheng et al. [40] presented a model constructed by the combination of the social force model (Helbing et al., 1995) and neural network, to simulate collective behaviors of pedestrians in various situations. Seyfried et al. [41] introduced a modified social force model (Helbing et al., 1995) to simulate pedestrian dynamics and qualitatively analyze the influence of various approaches for the interaction between the pedestrians on the resulting velocity–density relation. Parisi and Dorso [42] applied the social force model (Helbing et al., 2000) to study the evacuation from the room with an exit; later they modified slightly the social force model to simulate the evacuation of pedestrians from a room with one exit under panic [43].

Furthermore, the social force models are combined with other models to study crowd evacuation. Lin et al. [44] presented an agent-based system for crowd evacuation in an emergency

situation based on the social force mode (Helbing et al., 2000). Guo and Huang [45] presented a mobile lattice gas model based on the social force model (Helbing et al., 2000) to simulate the pedestrian evacuation process in a public building. The model can capture the basic characteristics of pedestrian evacuation such as arching and clogging behavior. Additionally, the two-dimensional cellular automata model proposed by Yang et al. [1] (depicted in Section 2.1), the multi-grid model proposed by Song et al. [37] (depicted in Section 2.2) are both based on the social force model.

For comparison, Table 4 shows features of these social force models.

2.4. Fluid-dynamic models

Pedestrian crowds have been described with fluid-like properties over the last decades. Henderson [46] has conjectured that pedestrian crowds behave similarly to gases or fluids. Bradley [47] has hypothesized that the Navier–Stokes equations governing fluid motion could be used to describe motion in crowds at very high densities. Helbing et al. [48] have summarized that at medium and high densities, the motion of pedestrian crowds shows some striking analogies with the motion of fluids. For example, the footprints of pedestrians in snow look similar to streamlines of fluids or, again, the streams of pedestrians through standing crowds are analogous to riverbeds.

Fluid-dynamic models describe how density and velocity change over time with the use of partial differential equations. Hughes [49] studied the choice of path by using a continuum model based on well-defined observations of pedestrian behavior. In order to understand the mechanics of pedestrian crowd motion in especially large crowds, Hughes [50] derived the equations of motion governing the two-dimensional flow of pedestrians. To further understand crowd in motion, Hughes [51] developed a continuum model to describe crowds as “thinking fluids” based on well-defined hypotheses. Furthermore, Colombo and Rosini [52] presented a continuum model for pedestrian flow to describe typical features of this kind of flow such as some effects of panic. In particular, this model describes the possible overcompressions in a crowd and the fall in the outflow through a door of a panicking crowd jam. They considered the situation where a group of people needs to leave a corridor through a door. If the maximal outflow allowed by the door is low, then the transition to panic in the crowd approaching the door may likely cause a dramatic reduction in the actual outflow, decreasing the outflow even more.

For comparison, Table 5 shows features of these fluid-dynamic models.

2.5. Agent-based model

ABMs are computational models [53] that build social structures from the “bottom-up”, by simulating individuals with virtual agents, and creating emergent organizations out of the

Table 4
Features of these social force models

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Typical phenomena
[38]	SF	Homogeneous	Microscopic	Continuous	Normal	Lane formation, oscillatory change
[39]	SF	Homogeneous	Microscopic	Continuous	Emergency	Clogging, faster-is-slower, mass behavior
[40]	SF	Homogeneous	Microscopic	Continuous	Normal/emergency	Crowd impatience
[41]	SF	Homogeneous	Microscopic	Continuous	Normal	–
[42]	SF	Homogeneous	Microscopic	Continuous	Emergency	Faster-is-slower, clogging
[43]	SF	Homogeneous	Microscopic	Continuous	Emergency	Faster-is-slower
[44]	SF and AB	Homogeneous	Microscopic	Continuous	Emergency	Block, mass behavior
[45]	SF and LG	Homogeneous	Microscopic	Discrete	Emergency	Arching, clogging

Table 5
Features of these fluid-dynamic models

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Typical phenomena
[50]	FD	Homogeneous	Macroscopic	Continuous	Normal	Fluid behavior
[51]	FD	Homogeneous	Macroscopic	Continuous	Normal	“Thinking fluids” behavior
[52]	FD	Homogeneous	Macroscopic	Continuous	Emergency	Overcompression effect

Table 6
Features of these agent-based models

Source	Approaches	Individuals/groups	Scale	SSFm	Situations	Typical phenomena
[55]	AB	Heterogeneous	Microscopic	Continuous	Emergency	Rule-based behavior
[56]	AB and SF	Heterogeneous	Microscopic	Continuous	Emergency	Decision behavior, escape behavior, risk behavior
[57]	AB and SF	Heterogeneous	Microscopic	Continuous	Normal/emergency	Queuing, pushing behavior, panic propagation, impatience
[58]	AB and CA	Heterogeneous	Microscopic	Discrete	Normal/emergency	Lane formation herding behavior, obstacle avoidance behavior
[59]	AB and CA	Heterogeneous	Microscopic	Discrete	Normal	Competitive and collaborative behaviors
[60]	AB and CA	Heterogeneous	Microscopic	Discrete	Emergency	Decision behavior
[61]	AB	Heterogeneous	Microscopic	Discrete	Emergency	Competitive behavior, queuing behavior, herding behavior.

operation of rules that govern interactions among agents. Bonabeau [54] supported the following point of view: in agent terms, collective panic behavior is an emergent phenomenon that results from relatively complex individual-level behavior and interactions among individuals; the ABM seems ideally suited to provide valuable insights into the mechanisms and preconditions for panic and jamming by incoordination.

In the last few years, the ABM technique has been used to study crowd evacuation in various situations. ABMs are generally more computationally expensive than cellular automata, lattice gas, social force or fluid-dynamic models. However, their ability to allow each pedestrian to have unique behaviors makes it much easier to model heterogeneous humans. For example, ZARBOUTIS and MARMARAS [55] presented an ABM to simulate a metro system in the case of a tunnel fire and to explore the effects that individual actions and their interdependence can have on the performance of the whole system. The two experiments demonstrated that the simulation may support the search for efficient rescue plans. BRAUN et al. [56] presented an ABM based on the social force model (Helbing et al., 2000) to simulate impacts of different floors, walls, and obstacles on agents, and interactions among agents in emergency situations. PELECHANO et al. [57] presented a multi-agent model called the **High-Density Autonomous Crowds (HiDAC) model**. HiDAC is a parameterized social force model that depends on psychological and geometrical rules. It can be tuned to simulate different types of crowds, ranging from extreme panic situations (fire evacuation) to high-density crowds under calm conditions (leaving a cinema after a movie). TOYAMA et al. [58] proposed an ABM based on cellular automata, which represented different pedestrian characteristics such as gender, speed, room geometry knowledge, herding behavior, and obstacle avoidance behavior. BANDINI et al. [59] designed heterogeneity into the Situated Cellular Agent (SCA) model and applied it to simulate crowd behavior in an underground station. HENEIN and WHITE [60] proposed the Swarm Information Model to study crowds of heterogeneous individuals who base their actions on the differing perceptions of the world engendered by unfolding information. Additionally, PAN et al. [61] presented a multi-agent-based framework to simulate human and social behaviors during emergency evacuations. In the system, human individuals are modeled by a certain degree of heterogeneity. During simulations of human behavior, the system can reproduce emergent human

social behaviors such as competitive behavior, queuing behavior, and herding behavior.

For comparison, Table 6 shows features of these ABMs.

2.6. Game theoretic models

If the interactive decision process of the evacuees is rational, a game theoretic approach can be adopted to model the decision situation [62]. In a game, the evacuees assess all of the available options and select the alternative that maximizes their utility. Each evacuee's final utility payoffs will depend on the actions chosen by all evacuees. The interactive situation, specified by the set of participants, the possible courses of action of each participant, and the set of all possible utility payoffs, is called a game.

For one exit, the competitive behavior of the pedestrians in emergency egress could be interpreted in a game theoretical way [13]. For several exits, LO et al. [62] established a non-cooperative game theory model for the dynamic exit selection process of evacuees. The model examines how the rational interacting behavior of the evacuees will affect the evacuation patterns. For the exit selection process, a mixed strategy is considered as the probability of exit choice. The mixed strategy Nash Equilibrium for the game describes the equilibrium for the evacuees and the congestion states of exits.

Table 7 shows features of the game theoretic model.

2.7. Approaches based on experiments with animals

The use of animals is a new approach for studying crowd evacuation. Experiments in genuine escape panic are difficult, especially with humans because of possible ethical and even legal concerns. The dynamics of escape panic are not completely understood because studies have been largely confined to numerical simulations [63].

SALOMA et al. [63] studied the dynamics of escape panic in mice escaping from a water pool to a dry platform through an exit door. The experiment showed the behavior of panicking groups and how it is influenced by the architecture of the space in which they are confined. The experimental results revealed that for a critical sampling interval their escape behaviors agreed with the numerically predicted exponential and powerlaw frequency

Table 7
Features of the game theoretic model

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Typical phenomena
[62]	Game theory	Homogeneous	Microscopic	Discrete	Emergency	Exit selection

Table 8
Features of these experiments with animals

Source	Approaches	Individuals/groups	Scale	SAT	Situations	Phenomena
[63]	Experiments with mice	Homogeneous	Microscopic	–	Emergency	Self-organized queuing, scale-free behavior in escape panic
[64]	Experiments with ants	Homogeneous	Microscopic	–	Emergency	Panic propagation, herding behavior

distributions of the exit burst size even for short time durations. Altshuler et al. [64] noted that panic propagated rapidly by imitation. By using ants as pedestrians, they demonstrated the validity of such prediction through experiments in which low-panic and high-panic scenarios were examined. For example, when individuals under panic try to escape from a room with two symmetrically located exits, one of the exits is more heavily used than the other. The experimental findings coincide well with the theoretical predictions reported by Helbing et al. (2000) for humans, and suggest that some features of the collective behavior of humans and ants are quite similar when escaping under panic.

Table 8 shows features of these experiments with animals.

3. Discussion

As mentioned earlier, observed phenomena occurring during evacuations have been reproduced by seven approaches in the last few years. For example, panic escape has been reproduced by the social force model [39] and the animal experiments [63,64]; following the crowd (herding behavior, i.e., doing what others do) has been simulated by the social force model [39] and the ABM [58]; “faster-is-slower” has been mimicked by the social force models [39,42,43] and the CAFE [22]; congestion has been presented by the cellular automata models [8,10,20], the lattice gas models [26–28], the fluid-dynamic model [52] and a game theoretic model [62], etc. However, all these approaches obey some defined sets of rules under certain restricted conditions, which have inherent advantages and disadvantages.

Approaches based on lattice gas and cellular automata are typical models in evacuation modeling. These models are discrete in space, time and state variables. This makes the models ideally suited for large-scale computer simulations. Moreover, cellular automata models have strong expressive power to represent many collective behaviors. For example, Burstedde et al. [65] have proposed a stochastic cellular automata model to simulate pedestrian behavior, where the key mechanism is the floor field that acts as a substitute for pedestrian intelligence. The introduction of such a floor field is sufficient to model collective effects (e.g., lane formation and oscillations at doors) and self-organization encountered in pedestrian dynamics. Perez et al. [9] have also proposed a cellular automata model that simulates the exit dynamics of pedestrians in a room, where prominent features of emergent evacuation (e.g., arching and the phenomenon similar to the “faster-is-slower” effect) can be reproduced.

What is more, cellular automata models achieve very good simulation results. For example, Moroshita and Shiraishi [66] simulated the pedestrian flow in the concourse of Tamachi Station, an average size station in Tokyo Metropolitan area, with

the use of the cellular automata model in order to propose an evaluation index for billboards set along the pathway in the concourse. The number of people in the concourse was counted in practice and it showed good agreement with the field investigation database. Additionally, lattice gas models also successfully describe some phenomena during crowd evacuations. Nagai et al. [33] investigated the dynamical behavior of walkers and crawlers evacuating from the channel through the exit by the experiments and the lattice gas simulation. Especially, they studied the dependence of the mean flow rate on the density as well as the dependence of the flow rate on the exit width for walkers and crawlers. The experimental results are consistent with the simulation ones. In the experiments and the lattice gas simulations, Nagai et al. [29], and Isobe et al. [36] also obtained the consistent results as obtained earlier.

Contact forces in crowd panic may generate high pressures that can asphyxiate people in the crowd and even push down a brick wall. However, either cellular automata models or lattice gas models cannot properly take these high-pressure characteristics into account, while social force models can [42,43]. For example, in Helbing’s simulation [39], above a desired velocity of about $v_0 = 5$ m/s, people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of 1600 N/m. Furthermore, some prominent effects (e.g., stumbling, falling, injury, ache, and others walking over the fallen pedestrians, etc) that can appear during an emergency evacuation are also ignored in the models.

Evacuation simulation using the social force models proposed by Helbing et al. [38,39] has led to great progress in the field of crowd evacuation. The models based on social forces can successfully simulate the most typical phenomena observed in pedestrian dynamics and achieve very realistic simulation results. For example, Moldovan et al. [67] investigated the propagation and dispersion of pedestrian pulses in the underground stations of Buenos Aires. The flow rate curves were collected by two synchronized and separated cameras. The curve measured by the first camera was used as the input of the simulated system, where pedestrian flow was mimicked by implementing the social force model. The output can be compared to the data obtained by the second camera. The simulation result exactly equals to the experimental one. Another example is that of Seyfried et al. [68] who modified the social force model (Helbing et al., 1995) to investigate the influence of the required length and remote action on the resulting velocity–density relation. The model-parameter can be adjusted to yield a good agreement with the empirical fundamental diagram.

The social force models can reproduce some observed behaviors of pedestrian flow such as “faster-is-slower”, arching

and clogging due to the interactions among pedestrians [39,42,43]. However, it was argued that some of the underlying model assumptions oversimplified the process of pedestrians' way finding through the traffic flow [69]. The models consist mainly of three terms, which correspond to the acceleration. These terms are (1) the desired velocity of motion, (2) the repulsive and attractive interactions with other pedestrians or obstacles, and (3) the interaction with walls and other rigid objects for a certain pedestrian moving toward the destination. Moreover, Henein and White [70] made an analysis of the social force models and supported the following idea: people are not grains. Additionally, a direct and invariant relationship between sensory perceptions of individuals (such as noticing a person ahead) and the responses engendered (e.g., repulsion inversely proportional to distance) deny the contributions of individual agents who make local decisions based on personal strategies. This abstracts away a very important attribute of crowds, namely, the effect of non-homogeneity.

Fluid-dynamic models use the analogy of fluid dynamics to describe the crowd flow in the form of partial differential equations. Therefore, these models are usually applied to simulate the jamming situations in which the crowd flow is dense. It is remarkable that these non-linear partial differential equations that govern the flow of a crowd of people are conformably mappable even in unsteady situations [51]. However, in these analogies, the fluid-dynamic equation is difficult and not flexible [71]. The main reason is that it is highly non-linear and set up based on several hypotheses [49–52]. Moreover, the use of fluid analogies is of limited value except at dangerously high-crowd densities ($>4\text{ m}^{-2}$) where (compressibility) pressure waves can occur [50].

All ABMs are microscopic models and are based on some elementary form of intelligence for each agent [71]. Bonabeau [54] summarized benefits of ABM over other modeling techniques as follows: (1) ABM captures emergent phenomena, (2) ABM provides a natural description of a system, and (3) ABM is flexible. Currently, ABMs describe successfully pedestrian behaviors. For example, Bandini et al. [59] applied the SCA model to mimic crowd behavior in an underground station. The execution and analysis of several simulations showed that the behavior of the agents in the environment is consistent with a realistic scenario. Another example is that Braun et al. [56] proposed an ABM based on the social force model to simulate crowd evacuations in emergency situations. The results measured are very consistent with the simulation ones. For instance, for mean velocity on stairs with traffic jams, the experimental and simulated results are 0.5 and 0.47 m/s, respectively.

Although ABM is beneficial from the advancement of cognitive science [53], it is not a simple task to model human agents' behavior. The models describing human agents' behavior may be from quite simple to complicated ones. For example, the ABM presented by Zarboutis and Marmaras [55] is a quite simple one. The evacuees can be modeled as flexible agents, whose transition between two consecutive states can be described by a set of simple rules. On the contrary, in the MASSEgress framework proposed by Pan et al. [61], human cognitive processes are simulated by a "perception-action" model, in which an agent continuously assesses or "senses" the surrounding environment and makes decisions based on its decision model in a proactive fashion. In terms of control theory, this is a single-loop negative feedback system. In the framework, there are thousands of heterogeneous agents who interact. Therefore, from an effective perception of their environment until the execution of an action, multi-loop or multiple negative feedback systems might intervene. Thereby, modeling of such agents is a matter of highly sophisticated cognitive models.

A new approach to studying crowd evacuations is based on game theory. In a game theoretic model, the behavior of evacuees (players) is mimicked. The individual behavior in a crowd strongly depends on the behavior of other individuals in the crowd. For exit selection during evacuation process, the game theoretic approach can rationalize the interaction of the evacuees with the environment [62]. Therefore, this approach may be ideally suited to analysis of human reasoning and strategic thinking in an evacuation. However, the outcome of a game is characterized by a payoff matrix. It is difficult to identify an appropriate payoff matrix because of the large number of "players".

The models based on the above approaches are computational models, which can give numerical simulations. On the contrary, the approaches based on experiments with animals can achieve "real" simulations. Experiments with animals can reproduce some of the behaviors of panicking groups so that realistic crowd evacuation can be understood very easily. For example, escaping behaviors of mice and ants under panic reproduce those provided by observation from the real escape panic of pedestrians [63,64].

However, human beings, who have strong social consciousness, are greatly different from mice and ants in experiments. For example, social behaviors of evacuees, such as kin behavior [1] and decision-making for exit selection in an emergency [62], result from individual characteristics related to social consciousness (e.g., emotions, individual experience, and knowledge). It is impossible to reproduce these behaviors by means of animal experiments.

4. Conclusions and further research

With these factors (e.g., building shapes around the crowd, interactions among evacuees and their psychology) considered, crowd behaviors during an evacuation are very complex. Through analysis of previous studies on crowd evacuation, we have developed several conclusions.

Firstly, a new trend highlights a need to combine various approaches to study crowd evacuation, such as the cellular automata model based on lattice gases [23], the cellular automata models based on social force [1,22], the lattice gas model based on social force [37], the agent-based models (ABMs) based on cellular automata [58–60], the ABMs based on social force [56,57], etc. Because of pedestrian behavioral complexity and computer resource limitation, a pedestrian model is developed based on the advantages of various approaches. Generally speaking, a new model is produced by combination of basic principle of one approach with some rules of another approach. For example, Song et al. [37] developed the multi-grid model based on the lattice gas model and the social force model. A lattice gas model is simple and can achieve a high computational efficiency. A social force model is good at describing the interactions of pedestrians. Thus the multi-grid model can simulate the interactions among pedestrians efficiently. However, not all approaches can be combined to simulate crowd evacuations. For example, in all microscopic models (e.g., cellular automata, lattice gas, social force, and ABMs), the pedestrian is modeled as a particle. However, in the macroscopic models (e.g., fluid-dynamic models), a crowd of pedestrians is modeled as a fluid. Particle motions are different from fluid motions. Therefore, the microscopic approaches can be combined with each other to describe microscopic phenomena produced by pedestrians. According to the available literatures, the microscopic approaches and the macroscopic ones cannot be combined to model pedestrian behavior.

Secondly, the pedestrians in most models (experiments) are almost uniformly modeled although some researchers have considered the pedestrians as heterogeneous individuals or

groups in their models. However, in the real world, the crowd evacuation is a complex system composed of different pedestrians and environments. These pedestrians have various psychological states and physiological characteristics. In an evacuation, they interact and are affected differently by the environments around them.

Thirdly, some models have been introduced with human elements reflecting pedestrian characteristics, such as kin behavior based on the cellular automata model [1], escape panic based on the social force model [39], the “thinking fluids” based on the fluid-dynamic model [51], etc. Therefore, the phenomena reproduced by these human-like models are closer to those in a genuine evacuation.

Therefore, in further research, various approaches should be combined to study crowd evacuation. This research should concentrate on incorporating psychological and physiological factors into the evacuation models (experiments) reproducing individual and collective behaviors observed in real evacuations. In this way, the results of the models (experiments) will be quite similar to the genuine situations arising during evacuations.

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